

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS
AS LEVEL**

H230/02

MATHEMATICS A

Pure Mathematics and Mechanics

WEDNESDAY 23 MAY 2018: Morning

**TIME ALLOWED: 1 hour 30 minutes
plus your additional time allowance**

MODIFIED ENLARGED

YOU MUST HAVE:

**Printed Answer Booklet sent with the standard paper or
any suitable paper provided by the centre. The Printed
Answer Booklet may be enlarged by the centre.**

YOU MAY USE:

a scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF



INSTRUCTIONS

Use black ink. HB pencil may be used for graphs and diagrams only.

Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number or write them on the paper provided.

Answer ALL the questions.

IF YOU USE THE PRINTED ANSWER BOOKLET WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

The total number of marks for this paper is 75.

The marks for each question are shown in brackets [].

YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.

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Formulae

AS Level Mathematics A (H230)

Binomial series

$$(a + b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n$$

$$(n \in \mathbb{N}), \text{ where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\sum (x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

$$\text{If } X \sim B(n, p) \text{ then } P(X = x) = \binom{n}{x} p^x (1-p)^{n-x},$$

Mean of X is np , Variance of X is $np(1-p)$

Kinematics

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

SECTION A: Pure Mathematics
Answer ALL the questions

- 1** In triangle ABC , $AB = 20$ cm and angle $B = 45^\circ$.
- (i) Given that $AC = 16$ cm, find the two possible values for angle C , correct to 1 decimal place. [4]
- (ii) Given instead that the area of the triangle is $75\sqrt{2}$ cm², find BC . [2]
- 2** (i) The curve $y = \frac{2}{3+x}$ is translated by four units in the positive x -direction. State the equation of the curve after it has been translated. [2]
- (ii) Describe fully the single transformation that transforms the curve $y = \frac{2}{3+x}$ to $y = \frac{5}{3+x}$. [2]
- 3** In each of the following cases choose one of the statements

$$P \Rightarrow Q \quad P \Leftarrow Q \quad P \Leftrightarrow Q$$

to describe the relationship between P and Q .

(i) $P: y = 3x^5 - 4x^2 + 12x$

$$Q: \frac{dy}{dx} = 15x^4 - 8x + 12 \quad [1]$$

(ii) $P: x^5 - 32 = 0$ where x is real

$Q: x = 2$ [1]

(iii) $P: \ln y < 0$

$Q: y < 1$ [1]

4 (i) Express $4x^2 - 12x + 11$ in the form $a(x + b)^2 + c$. [3]

(ii) State the number of real roots of the equation

$$4x^2 - 12x + 11 = 0. \text{ [1]}$$

(iii) Explain fully how the value of r is related to the number of real roots of the equation

$$p(x + q)^2 + r = 0 \text{ where } p, q \text{ and } r \text{ are real constants and } p > 0. \text{ [2]}$$

5 In this question you must show detailed reasoning.

The line $x + 5y = k$ is a tangent to the curve

$$x^2 - 4y = 10. \text{ Find the value of the constant } k. \text{ [5]}$$

- 6 A pan of water is heated until it reaches 100°C . Once the water reaches 100°C , the heat is switched off and the temperature $T^{\circ}\text{C}$ of the water decreases. The temperature of the water is modelled by the equation**

$$T = 25 + ae^{-kt},$$

where t denotes the time, in minutes, after the heat is switched off and a and k are positive constants.

- (i) Write down the value of a . [1]**
- (ii) Explain what the value of 25 represents in the equation $T = 25 + ae^{-kt}$. [1]**

When the heat is switched off, the initial rate of decrease of the temperature of the water is 15°C per minute.

- (iii) Calculate the value of k . [3]**
- (iv) Find the time taken for the temperature of the water to drop from 100°C to 45°C . [3]**
- (v) A second pan of water is heated, but the heat is turned off when the water is at a temperature of less than 100°C . Suggest how the equation for the temperature as the water cools would be modified by this. [1]**

7 (i) Show that the equation

$$2\sin x \tan x = \cos x + 5$$

can be expressed in the form

$$3\cos^2 x + 5\cos x - 2 = 0. \text{ [3]}$$

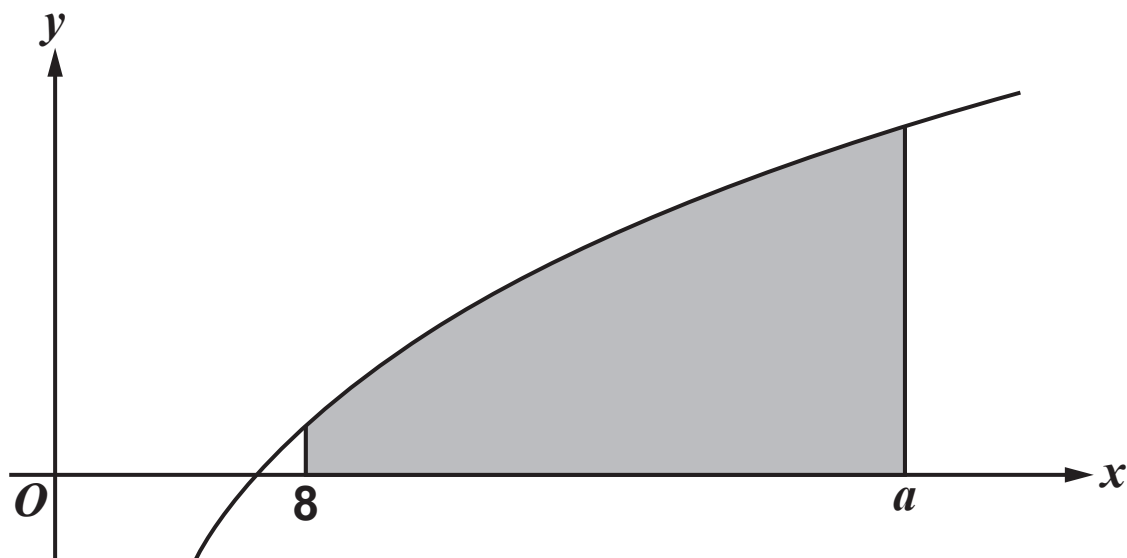
(ii) Hence solve the equation

$$2\sin 2\theta \tan 2\theta = \cos 2\theta + 5,$$

giving all values of θ between 0° and 180° , correct to 1 decimal place. [5]

8 In this question you must show detailed reasoning.

The diagram below shows part of the graph of $y = 2x^{\frac{1}{3}} - \frac{7}{x^{\frac{1}{3}}}$. The shaded region is enclosed by the curve, the x -axis and the lines $x = 8$ and $x = a$, where $a > 8$.



Given that the area of the shaded region is 45 square units, find the value of a . [9]

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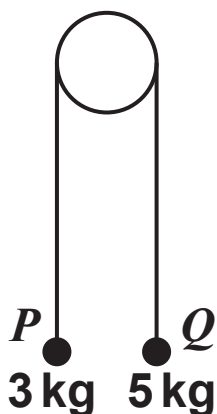
SECTION B: Mechanics
Answer ALL the questions

- 9** In this question the horizontal unit vectors \mathbf{i} and \mathbf{j} are in the directions east and north respectively.

A model ship of mass 2 kg is moving so that its acceleration vector $a\text{ m s}^{-2}$ at time t seconds is given by $\mathbf{a} = 3(2t - 5)\mathbf{i} + 4\mathbf{j}$. When $t = T$, the magnitude of the horizontal force acting on the ship is 10 N .

Find the possible values of T . [4]

- 10 Particles P and Q , of masses 3 kg and 5 kg respectively, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The system is held at rest with the string taut. The hanging parts of the string are vertical and P and Q are above a horizontal plane (see diagram below).

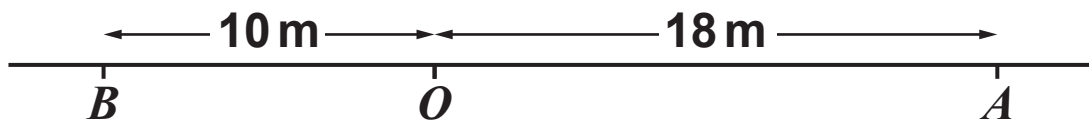


- (i) Find the tension in the string immediately after the particles are released. [4]

After descending 2.5 m , Q strikes the plane and is immediately brought to rest. It is given that P does not reach the pulley in the subsequent motion.

- (ii) Find the distance travelled by P between the instant when Q strikes the plane and the instant when the string becomes taut again. [4]

- 11 A particle P is moving along a straight line with constant acceleration. Initially the particle is at O . After 9 s, P is at a point A , where $OA = 18$ m (see diagram below) and the velocity of P at A is 8 m s^{-1} in the direction \overrightarrow{OA} .



- (i) (a) Show that the initial speed of P is 4 m s^{-1} . [2]

- (b) Find the acceleration of P . [2]

B is a point on the line such that $OB = 10$ m, as shown in the diagram.

- (ii) Show that P is never at point B . [4]

A second particle Q moves along the same straight line, but has variable acceleration. Initially Q is at O , and the displacement of Q from O at time t seconds is given by

$$x = at^3 + bt^2 + ct,$$

where a , b and c are constants.

It is given that

the velocity and acceleration of Q at the point O are the same as those of P at O ,

Q reaches the point A when $t = 6$.

- (iii) Find the velocity of Q at A . [5]

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